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Wave Propagation in a Magnetized Dusty Plasma

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The properties of waves propagating in a magnetized dusty plasma are analyzed for both high and low values of the ion-cyclotron frequency. The electrons are considered to be hot. The dispersion relation for a wave propagating through the dusty plasma at an arbitrary angle with the external magnetic field is derived. This rectifies some errors in the earlier treatment of Cramer and Vladimirov (1996). Effects of the presence of the dust on the characteristics of wave propagation are discussed.

1. INTRODUCTION

Studies of wave propagation in a dusty plasma have been found to be very interesting because of their importance in practical situations, such as those relating to the Earths environment (Goertz, 1989), space plasmas (de Angelis, 1992; Dhar, 1996) and laboratory experiments (Mendis and Rosenberg, 1994; Prabhuram and Goree, 1996; Barkman *et al.*, 1995; Chen, 1995). A dusty plasma is a low-temperature ionized gas whose constituents are electrons, ions, and micrometer-size dust grains. The dust particles get negatively charged due to the attachment of background electrons on the surface via collisions (Northrop, 1992; Dwivedi, 1993; Allen, 1992; Chow *et al.*, 1993). In particular charged dust grains collect electrons and ions from the background plasma. The presence of dust particles changes the plasma parameters and affects the collective process. For many dusty plasmas this charge is negative and of the order of $\sim 10^2 e$ to $10^3 e$ (e is the electron charge).

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Due to the heavy mass, the time of response of dust grains to high-frequency oscillations in the plasma is very large. Therefore, the presence of dust grains is in general of less consequence in high-frequency oscillation except for producing a damping (de Angelis *et al.*, 1989). In fact, the presence of charged dust grains gives rise to new kinds of modes called 'dust modes.' Rao *et al.* (1990) first theoretically investigated the propagation of the dust acoustic mode in a plasma. Subsequently several authors (D'Angelo, 1990; Shukla, 1992; Shukla and Silin, 1992; Rao, 1993; Varma *et al.*, 1993; Ma and Yu, 1996; Shchekinov, 1997; Amin *et al.*, 1997) studied different aspects of the dust acoustic mode in dusty plasmas.

On the other hand, the propagation of Alfvèn waves and magnetosonic waves through a dusty plasma are important and have been discussed by several authors (Philip *et al.*, 1987; Salimullah, 1996; Salimullah and Amin, 1996; Das *et al.*, 1996). Cramer and Vladimirov (1996) studied the propagation of waves through a dusty plasma at a frequency below and of the order of the ion-cyclotron frequency considering a nonzero electron temperature. They derived a general dispersion relation for waves propagating at an arbitrary angle with respect to the external magnetic field and showed that the presence of dust grains modifies the Alfvèn resonance absorption mechanism. In this communication we reconsider the problem studied by Cramer and Vladimirov (1996) and study the propagation of a wave in a magnetized dusty plasma with hot electrons and stationary dust grains. We derive the general dispersion relation of the wave at both high and low frequency propagating through the plasma making an angle θ with the direction of the magnetic field. We find that the behavior of propagating waves in the high- and low-frequency region turns out to be quite different than given by Cramer and Vladimirov (1996). The results are analyzed graphically.

2. FORMULATION

We consider a magnetized collisionless plasma consisting of electrons with density n_e, with equilibrium value n_{e0}, and ions with density n_i with equilibrium value n_{i0}. It is assumed that the usual hydrodynamic description is admissible. Then the equations of motion governing the plasma can be written as (Cramer and Vladimirov, 1996)

$$m_s \frac{\partial \mathbf{v}_s}{\partial t} + m_s(\mathbf{v}_s \cdot \nabla)\mathbf{v}_s = q_s[\mathbf{E} + (\mathbf{v}_s \times \mathbf{B})] - \frac{\nabla P_s}{n_s} \tag{1}$$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v}_s) = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \, \Sigma \, q_s(n_s \mathbf{v}_s) \tag{4}$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_o} \Sigma \, q_s n_s \tag{5}$$

$$\nabla \cdot \mathbf{B} = \mathrm{O} \tag{6}$$

where the subscript $s = e$ represents electrons and $s = i$ ions. Here (1) and (2) are the momentum and continuity equations for electrons and ions, respectively. Equations (3) to (6) are Maxwell's equations describing the interplay of the electric and magnetic fields and plasma parameters. The μ_o and ε_o represent, respectively, the permeability and dielectric constant of the medium. P_e is the electron pressure, given by $P_e = n_e K_B T_e$, where K_B is the Boltzmann constant. Note that equilibrium densities of electrons, ions, and dust particles are determined by the neutrality condition $en_{e0} + ez_d n_{d0} = en_{i0}$, where $n_{e,i,d}$ represent respectively the concentrations of electrons, ions, and dust in the plasma and ez_d is the charge of dust particles. The imbalance of equilibrium densities of electrons and ions $\delta = n_{e0}/n_{i0} \neq 1$ characterizes the presence of dust in the plasma; δ is less than unity in a dusty plasma, and $\delta = 1$ represents the dust-free plasma.

We consider a plane wave of the form $\exp\{i(k_x X + k_z Z - \omega t)\}$ to be propagating inside the dusty plasma. The background magnetic field B_0 is assumed to be in the z direction. Neglecting the electron inertia and assuming the ions are cold, we obtain the velocity and electric field components from the first-order equations of (1) and (6). The velocity components are

$$V_{ix} = \frac{i}{B_0(f^2 - 1)} (fE_x + iE_y) \tag{7a}$$

$$V_{iy} = \frac{1}{B_0(f^2 - 1)} (ifE_y + E_x) \tag{7b}$$

$$V_{iz} = \frac{i}{B_0 f} E_z \tag{7c}$$

$$V_{ex} = E_y/B_0 \tag{7d}$$

$$V_{ey} = 1/B_0\left(\frac{k_x}{k_z} E_z - E_x\right) \tag{7e}$$

$$V_{ez} = -\frac{k_x}{k_z B_0} E_y + \frac{i\omega e}{k_z^2 T_e K} E_z \tag{7f}$$

The transverse electric field components are

$$\begin{aligned}
\frac{E_y}{E_x} = & \left(i\omega\Omega_i \left\{ -\frac{\delta\omega^2 k_x^2 V_s^2}{\Omega_i^2} + \left[1 - \delta\left(1 - \frac{\omega^2}{\Omega_i^2}\right)\right]\left[(\omega^2 - V_s^2 k_z^2)\right.\right.\right. \\
& \left.\left.\left. \times \left(1 - \frac{\varepsilon_0 \mu_0 \omega^2}{k_z^2}\right) - V_A \frac{(\mu_0 \rho_i)^{1/2} \varepsilon_0 \omega^2 V_s^2}{e\Omega_i n_{i0}} (k_x^2 + k_z^2)\right]\right\}\right) \\
& \times \left([\omega^2 - V_A^2 (k_x^2 + k_z^2)(1 - \omega^2/\Omega_i^2)] \left\{ (\omega^2 - v_s^2 k_z^2) \right.\right. \\
& + \frac{\varepsilon_0 \omega v_s^2}{e\Omega_i} \left[\frac{\mu_0 e \omega \Omega_i}{k_z^2 V_s^2} (k_z^2 V_s^2 - \omega^2) \right. \\
& \left.\left. + \frac{V_A (\mu_0 \rho_i)^{1/2}}{n_{i0}} \omega (k_x^2 + k_z^2) \right] \right\} - \delta\omega^2 V_s^2 k_x^2 \\
& \left. - \frac{\varepsilon_0 \omega^4 V_s^2 \delta^2 k_x^2 \mu_0}{k_z^2} \left(\frac{\omega^2}{\Omega_i^2} - 1 \right) \right)^{-1}
\end{aligned} \tag{8a}$$

and

$$\begin{aligned}
\frac{E_y}{E_x} = & \left(i\omega\Omega_i \frac{e\Omega_i^2}{\varepsilon_0 \delta V_s^2 \omega^2 (\omega^2 - \Omega_i^2)} \left\{ \left[1 - \delta\left(1 - \frac{\omega^2}{\Omega_i^2}\right)\right](\omega^2 - V_s^2 k_z^2) - \delta \frac{\omega^2 V_s^2 k_x^2}{\Omega_i^2} \right.\right. \\
& \left.\left. + \mu_0 \omega^2 \frac{\varepsilon_0 \delta \omega^2 V_s^2}{\Omega_i^2} + V_A \frac{(\mu_0 \rho_i)^{1/2} \varepsilon_0 \delta \omega^2 V_s^2}{e\Omega_i^3 n_{i0}} (\omega^2 - \Omega_i^2)(k_x^2 + k_z^2) \right\} \right) \\
& \times \left(\frac{1}{\omega^2 - \Omega_i^2} \left\{ \Omega_i^2 \left(-\frac{e k_x^2}{\varepsilon_0} + \mu_0 e \omega^2 \right) + (\omega^2 - \Omega_i^2) \mu_0 e \delta \omega^2 \right.\right. \\
& \left.\left. + \frac{e\Omega_i^2}{\varepsilon_0 \delta \omega^2 V_s^2} \left[\omega^2 - V_A^2 (k_x^2 + k_z^2) \left(1 - \frac{\omega^2}{\Omega_i^2}\right) \right] (\omega^2 - V_s^2 k_z^2) \right\} \right)^{-1}
\end{aligned} \tag{8b}$$

The electric field component parallel to the magnetic field is given in terms of the transverse components by

$$E_z = -\frac{V_s^2\omega}{\Omega_i}\frac{k_x k_z}{1-\omega^2/\Omega_i^2}$$
$$\times \frac{(\omega/\Omega_i)E_x + i[1-(\mu_0\omega^2\delta\varepsilon_0/k_z^2)(1-\omega^2/\Omega_i^2)]E_y}{(\omega^2-k_z^2V_s^2)-(\mu_0\varepsilon_0\omega^2/k_z^2)(\omega^2-k_z^2V_s^2)+V_A(\mu_0\rho_i)^{1/2}\varepsilon_0\omega^2V_s^2(k_x^2+k_z^2)/e\Omega_i n_{i0}} \tag{8c}$$

On the other hand, using the velocity components in the Maxwell equations, we get

$$D_{11}E_x + D_{12}E_y + D_{13}E_z = 0 \tag{9a}$$

$$D_{21}E_x + D_{22}E_y + D_{23}E_z = 0 \tag{9b}$$

$$D_{31}E_x + D_{32}E_y + D_{33}E_z = 0 \tag{9c}$$

where

$$D_{11} = \frac{fi}{f^2-1}\left(\frac{-ek_x^2}{\varepsilon_0} + \mu_0 e\omega^2\right) + \frac{B_0\omega i}{n_{i0}}(k_x^2 + k_z^2) \tag{10a}$$

$$D_{12} = -\left(\frac{1}{f^2-1} + \delta\right)\left(-\frac{e}{\varepsilon_0}k_x^2 + \mu_0 e\omega^2\right) + \frac{e\delta k_x^2}{\varepsilon_0} \tag{10b}$$

$$D_{13} = -\frac{iek_xk_z}{\varepsilon_0}\left(\frac{1}{f} - \frac{\omega\Omega_i}{k_z^2V_s^2}\right) \tag{10c}$$

$$D_{21} = -\frac{iek_xk_zf}{\varepsilon_0(f^2-1)} \tag{10d}$$

$$D_{22} = \frac{e}{\varepsilon_0}k_xk_z\left(\frac{1}{f^2-1} + \delta\right) + \frac{k_x}{k_z}\delta\left(-\frac{e}{\varepsilon_0}k_z^2 + \mu_0 e\omega^2\right) \tag{10e}$$

$$D_{23} = i\left(\frac{1}{f} - \frac{\omega\Omega_i}{k_z^2V_s^2}\right)\left(-\frac{e}{\varepsilon_0}k_z^2 + \mu_0 e\omega^2\right) + \frac{iB_0}{n_{i0}}(k_z^2 + k_x^2) \tag{10f}$$

$$D_{31} = \frac{k_z}{\delta k_x}\left(\frac{1}{f^2-1} + \delta\right) \tag{10g}$$

$$D_{32} = \frac{iV_A^2k_z}{\delta\omega\Omega_i k_x}\left[\frac{\omega\Omega_i f}{V_A^2(f^2-1)} + (k_x^2 + k_z^2)\right] \tag{10h}$$

$$D_{33} = -1 \tag{10i}$$

In addition, the ion cyclotron frequency $\Omega_i = eB_0/m_i$, $f = \omega/\Omega_i$, the Alfvèn speed is given by

$$V_A = \frac{B_0}{(\mu_0 m_i n_{i0})^{1/2}} \tag{11}$$

$V_s^2 = n_{i0}KTe/n_{e0}m_i$, and $\delta = n_{e0}/n_{i0}$ describes the relative abundance of dust in the plasma.

It is important to note that the two values of E_y/E_x and also of E_z given by (8a)–(8c) are different from the values obtained by Cramer and Vladimirov (1996). If μ_0 is assumed to be zero, then (8a) and (8c) become identical to the equations of Cramer and Vladimirov (1996), but since (8b) is different the dispersion relation turns out to be different also. The general dispersion relation is obtained from (9a)–(9c), which is quite complicated and can be written as

$$PQ - RS = 0 \tag{12}$$

where

$$P = \omega^2[1 - \delta(1 - f^2)](1 - \beta\alpha/f^2) - \delta\bar{k}^2\alpha\beta\omega^2 + \mu_0\varepsilon_0\omega^2 f^2 V_s^2\delta + \delta\varepsilon_0\mu_0 V_s^2\alpha(\bar{k}^2 + 1)\omega^2(f^2 - 1) \tag{13a}$$

$$Q = \omega^2\left[1 - \frac{\alpha(\bar{k}^2 + 1)}{f^2}(1 - f^2)\right]\left\{\omega^2\left(1 - \frac{\alpha\beta}{f^2}\right) + \varepsilon_0 V_s^2\omega^2 \times \left[\mu_0\alpha(\bar{k}^2 + 1) - \frac{\mu_0 f^2}{\alpha\beta}\left(1 - \frac{\alpha\beta}{f^2}\right)\right]\right\} - \frac{\delta\omega^4\bar{k}^2\alpha\beta}{f^2} - \varepsilon_0\omega^4\mu_0\bar{k}^2 V_s^2\delta^2(f^2 - 1) \tag{13b}$$

$$R = -\delta\bar{k}^2\alpha\beta\omega^2 + \omega^2[1 - \delta(1 - f^2)]\left[\left(1 - \frac{\alpha\beta}{f^2}\right)\left(1 - \frac{\varepsilon_0\omega^2\mu_0}{k_z^2}\right) - \mu_0\varepsilon_0 V_s^2\alpha(\bar{k}^2 + 1)\right] \tag{13c}$$

$$S = \omega^4\left[1 - \frac{\alpha(\bar{k}^2 + 1)}{f^2}(1 - f^2)\right]\left(1 - \frac{\alpha\beta}{f^2}\right) - \frac{\delta\bar{k}^2\alpha\beta\omega^4}{f^2} + \mu_0\omega^4\varepsilon_0\delta V_s^2[1 - \delta(1 - f^2)] \tag{13d}$$

Here $\bar{k} = k_x/k_z$, $\alpha = V_A^2 k_z^2/\Omega_i^2$, and $\beta = V_s^2/V_A^2$.

It is seen that the dispersion relation given by (12) is different from that of Cramer and Vladimirov (1996). In a dust-free plasma, the dispersion relation (12) becomes identical to (3.13.6) of Chakraborty (1997) and that of other authors.

3. RESULTS AND DISCUSSION

To analyze the nature of the wave propagation we consider two situations separately: $f \gg 1$ and $f \ll 1$. In the situation $f \gg 1$ equation (12) simplifies to

$$\bar{k}^4\{\alpha[2\alpha\beta - \mu_0\varepsilon_0 V_s^2 f^2(1 + \beta\delta^2)]\} + \bar{k}^2\{\beta[\alpha(3 + 4\alpha) - \delta^2 f^2] - \varepsilon_0\mu_0 V_s^2 f^2[\delta^2\beta + 2(1 + \alpha)]\} + \left\{(1 - \delta^2 f^2) \times \left(\beta - \frac{\mu_0\varepsilon_0 V_s^2 f^2}{\alpha}\right) + \alpha\beta(3 + 2\alpha) - \varepsilon_0\mu_0 V_s^2 f^2[2 + \alpha(1 - \beta\,\delta^2)]\right\} = 0 \tag{14}$$

On the other hand, for $f \ll 1$, we get

$$\bar{k}^4\,[\alpha\delta(\alpha - \delta f^2)] + \bar{k}^2[\alpha\delta(2\alpha - \delta f^2) - \delta^2(1 - \delta)f^4] + \{\delta[\alpha + (1 - \delta)f]\,[\alpha - (1 - \delta)f]\} = 0 \tag{15}$$

Each of the equations (14) and (15) is quadratic in $\bar{k}^2$, so we have in total four possible roots. The solutions of these equations are studied for various values of f and δ. We have also analyzed the modulus $|\bar{k}^2|$. The results of such an analysis are depicted in Figs. 1–5. In general the solution $\bar{k}^2$ turns out to be complex. So we have plotted the Re $\bar{k}^2$. In Fig. 1a we show the variation of Re $\bar{k}^2$ for $\delta = 0.3$ and $\delta = 0.5$. We observe that the value of Re $\bar{k}^2$ reaches a maximum near $f = 0.15$ and 0.19 for $\delta = 0.5$ and $\delta = 0.3$, respectively, when $\alpha = 0.01$, but then becomes negative and a real value of $\bar{k}$ may not be possible at all. A similar phenomenon is depicted in Fig. 1b for $\delta = 0.7$ and $\delta = 0.9$. The other hand, the imaginary part of $\bar{k}^2$ is plotted in Fig. 2. Interestingly, throughout the range of frequency considered here Im $\bar{k}^2$ remains negative. We also plot $|\bar{k}^2|$ in Fig. 3. Note that for the range $0 < f < 0.3$, there still exists a peak for the same value of f. In Fig. 4a for $f > 0.3$, the graphs for $\delta = 0.3$, 0.5, 0.7, 0.9 show the same behavior. Observe that the values of $|\bar{k}^2|$ get steeper as δ decreases, that is, as the amount of the dust increases. On the other hand, the peak value of Re $\bar{k}^2$ goes up as δ increases, that is, as the amount of dust decreases. From Figs. 1a–4a it is important to note that the cutoff frequency is not obtained in the low-frequency region, i.e., $f < 1$, when $\alpha = 0.01$. An interesting situation occurs when $f \ll 1$ and $\delta \ll 1$ (Fig. 4b), i.e., when there are more dust particles, but the frequency is very low. In this case, the variation of $\bar{k}^2$ is a straight-line behavior. To get the cutoff frequency, we have taken higher values of α ($\alpha = 0.5$) and plotted $\bar{k}^2$, Re $\bar{k}$, and Im $\bar{k}$ in Fig. 4c. In this case, the cutoff frequency is obtained for $\delta = 0.1$ and $\delta = 0.3$,

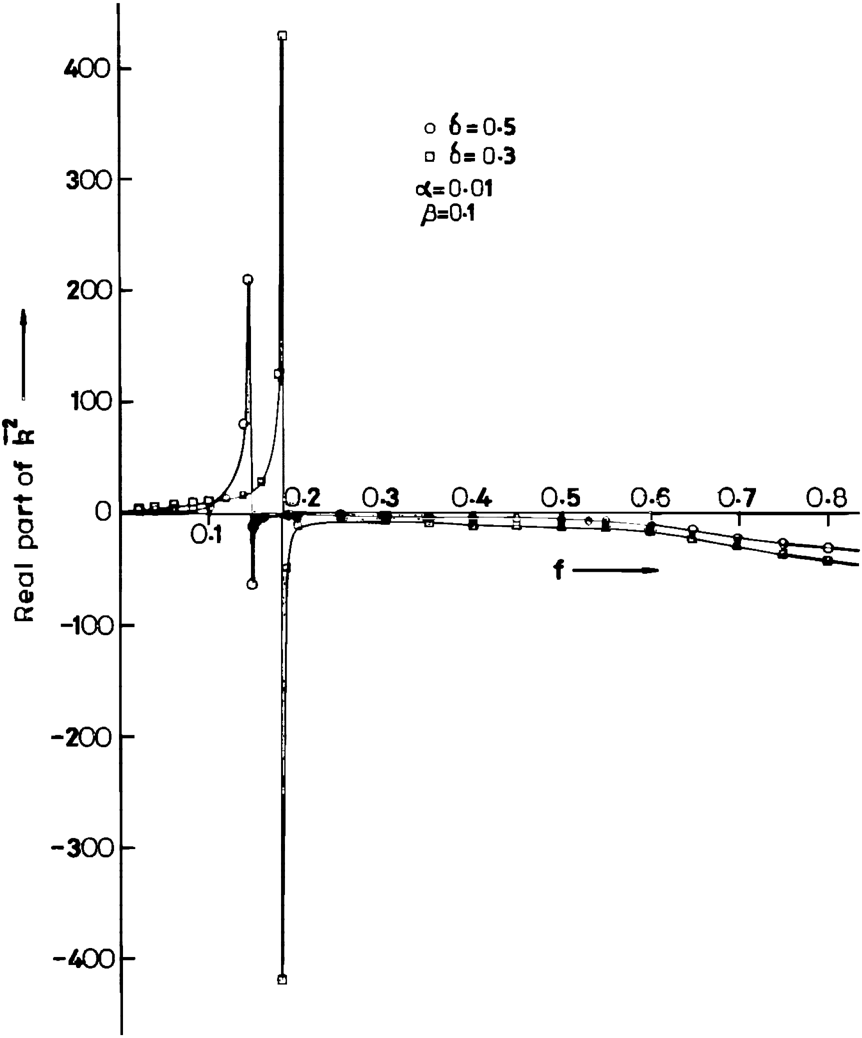


Fig. 1. (a) Real part of the normalized square of the perpendicular wave number $\bar{k} = k_x/k_z$ plotted against normalized frequency $f = \omega/\Omega_i$, for $f = 0.02–0.8$, $\delta = 0.3$ and 0.5, $\alpha = 0.01$, and $\beta = 0.1$.

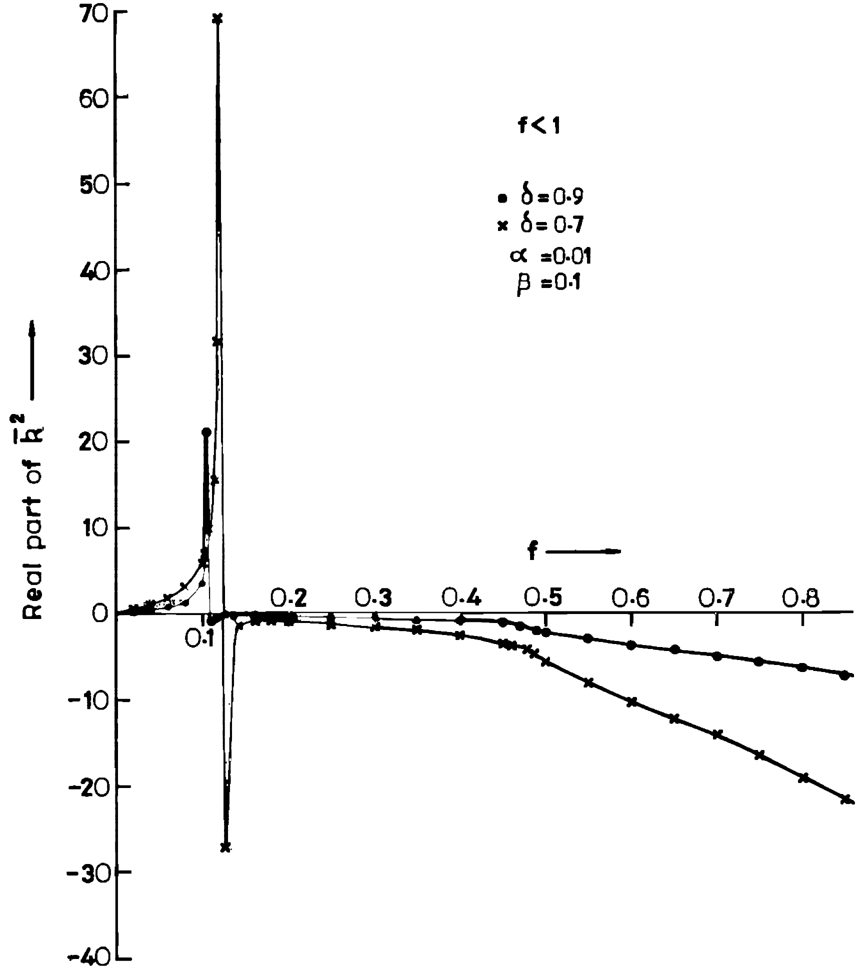


Fig. 1. Continued. (b) Real part of the normalized square of the perpendicular wave number $\bar{k}$ plotted against normalized frequency $f < 1$, for $f = 0.02$–0.8, $\alpha = 0.01$, and $\beta = 0.1$.

but no cutoff frequency is found for $\delta = 0.5$–0.9. In a different range of the values of f, i.e., when $f \gg 1$, the situation changes completely. We show $\bar{k}^2$ in Fig. 5 for $\alpha = 0.01$ and $\alpha = 0.5$. In each case the variation is along a straight line for the values $\delta = 0.1$–0.9. It is therefore observed from Figs. 4b and 5a–5c that the nature of the variation of $\bar{k}^2$ is the same in all the cases. In Figs. 4b, 5a, and 5c Im $\bar{k}^2$ is always negative. This means either $k_1 > 0$, $k_2 < 0$ or $k_1 < 0$, $k_2 > 0$, when $\bar{k} = k_1 + ik_2$. In the first case we have growing instability, but in the

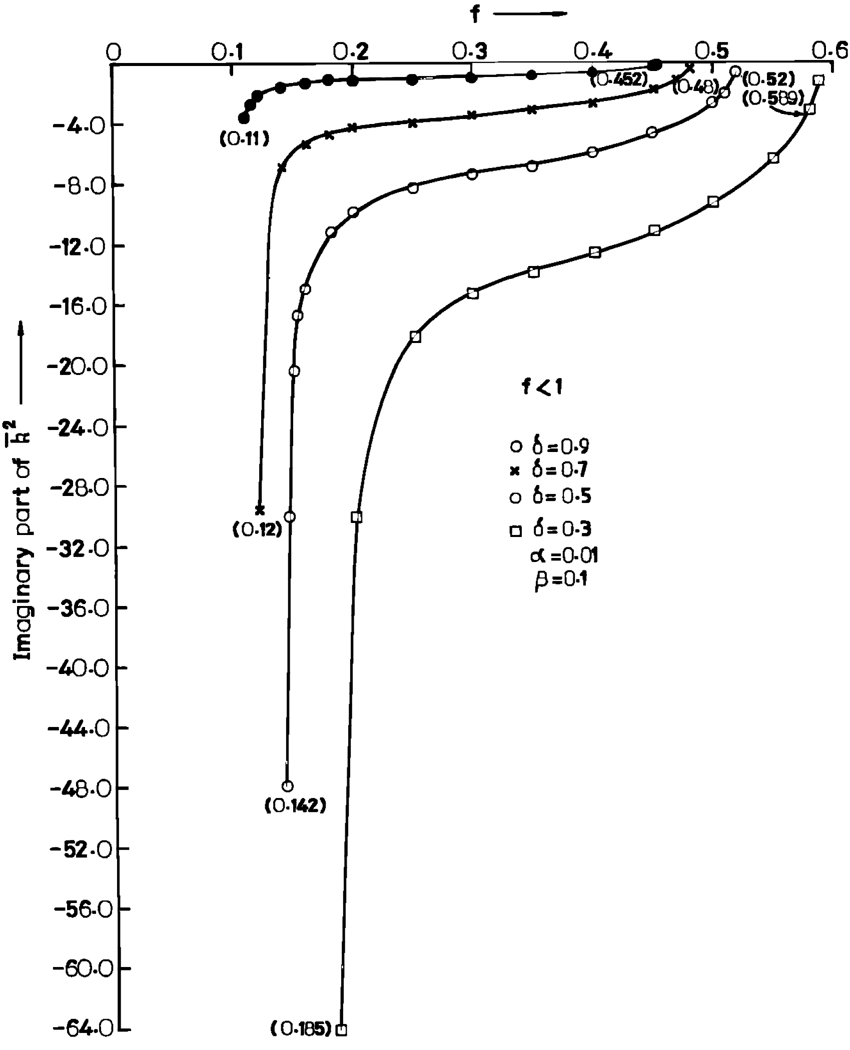


Fig. 2. Imaginary part of $\bar{k}^2$ plotted against normalized frequency f for $f = 0.1$–0.6, $\delta = 0.3$, 0.5, 0.7, and 0.9, $\alpha = 0.01$, and $\beta = 0.1$.

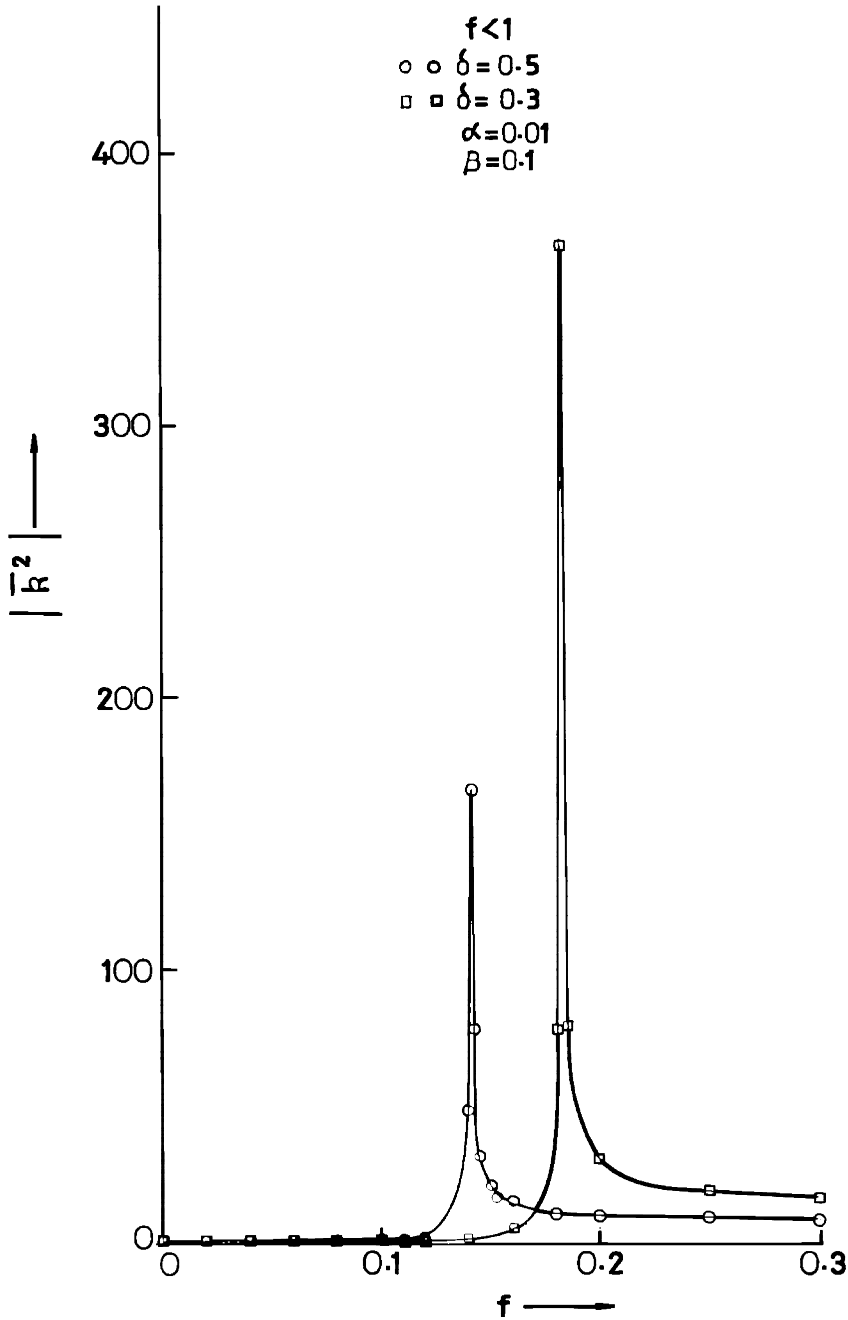
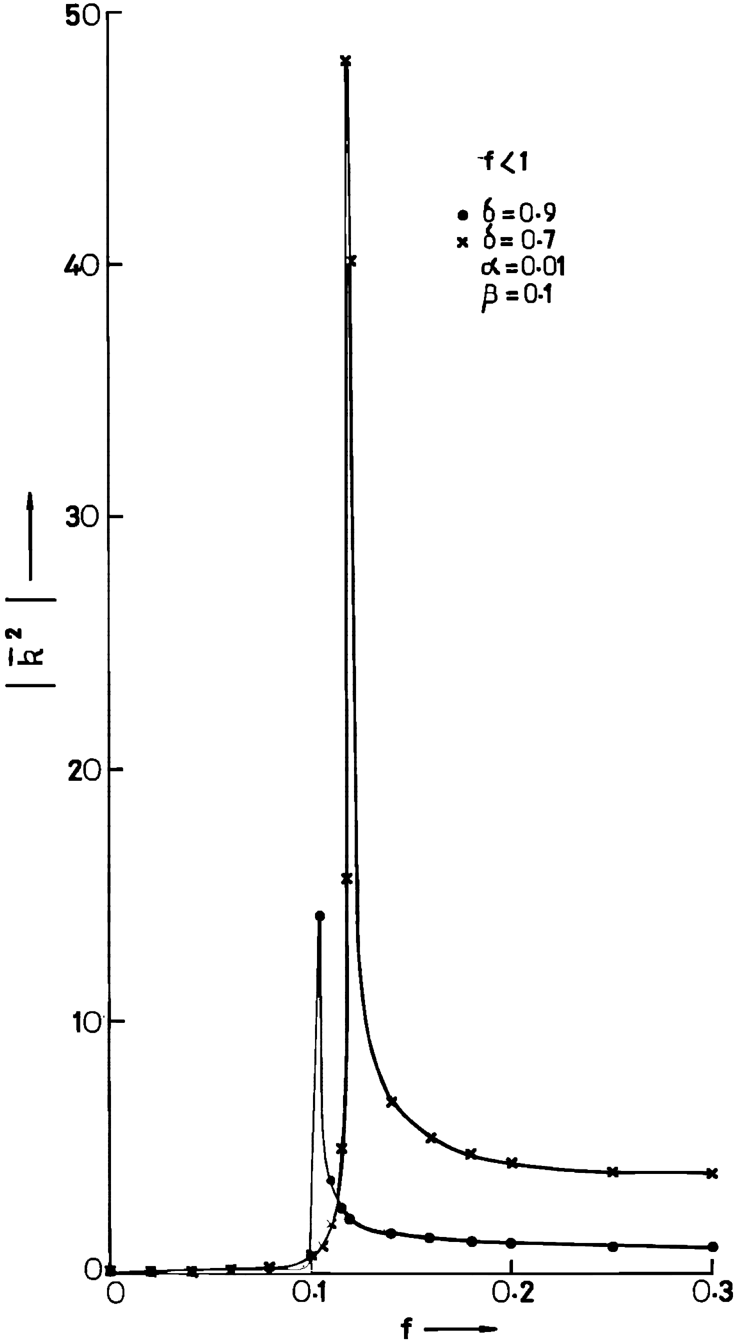


Fig. 3. Variation of $|\bar{k}^2|$ with the normalized frequency f for f = 0.02–0.3, α = 0.01, β = 0.01, and (a) δ = 0.3 and 0.5, (b) δ = 0.7 and 0.9.



f<1
● δ=0·9
× δ=0·7
α=0·01
β=0·1
50
40
30
20
10
0
|$\bar{R}^2$|
0
0·1
0·2
0·3
f

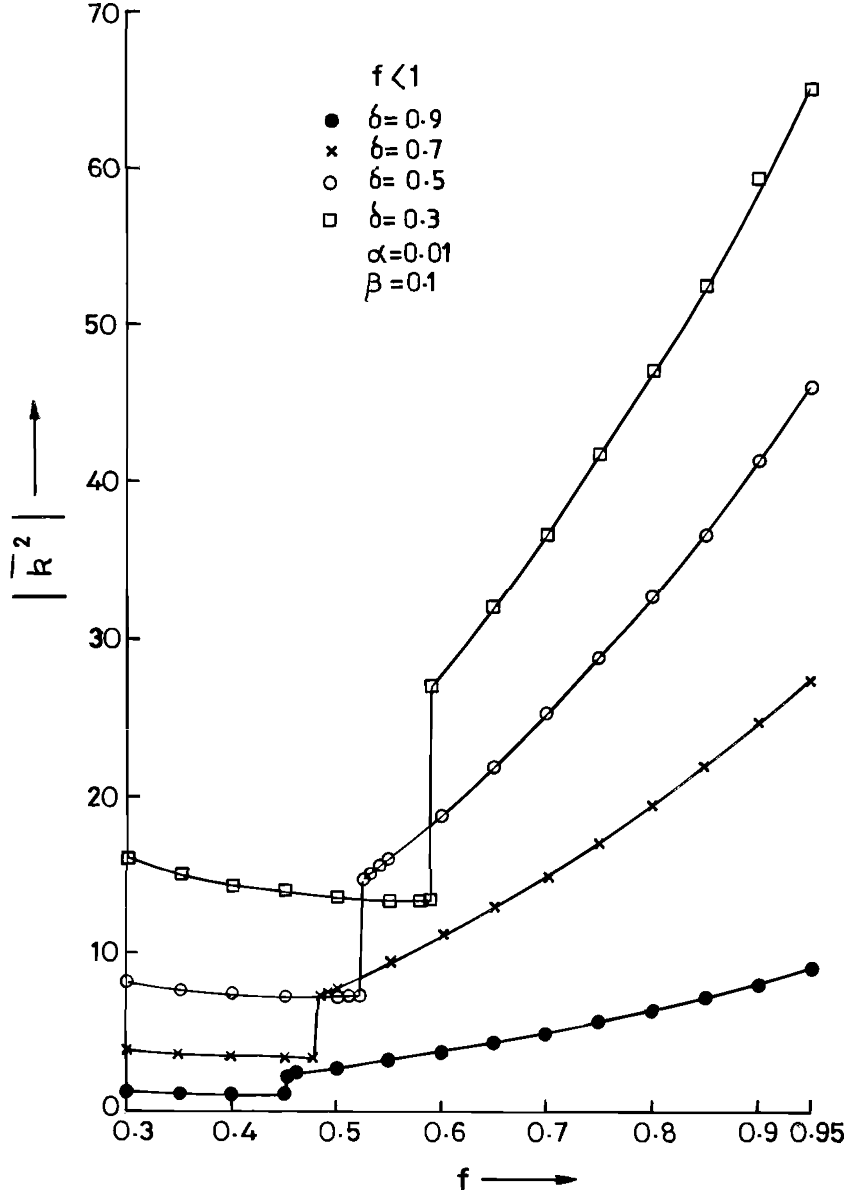


Fig. 4. (a) Variation of $|\bar{k}^2|$ with the normalized frequency f for $f = 0.3–0.95$, $\delta = 0.3–0.9$, $\alpha = 0.01$, and $\beta = 0.1$.

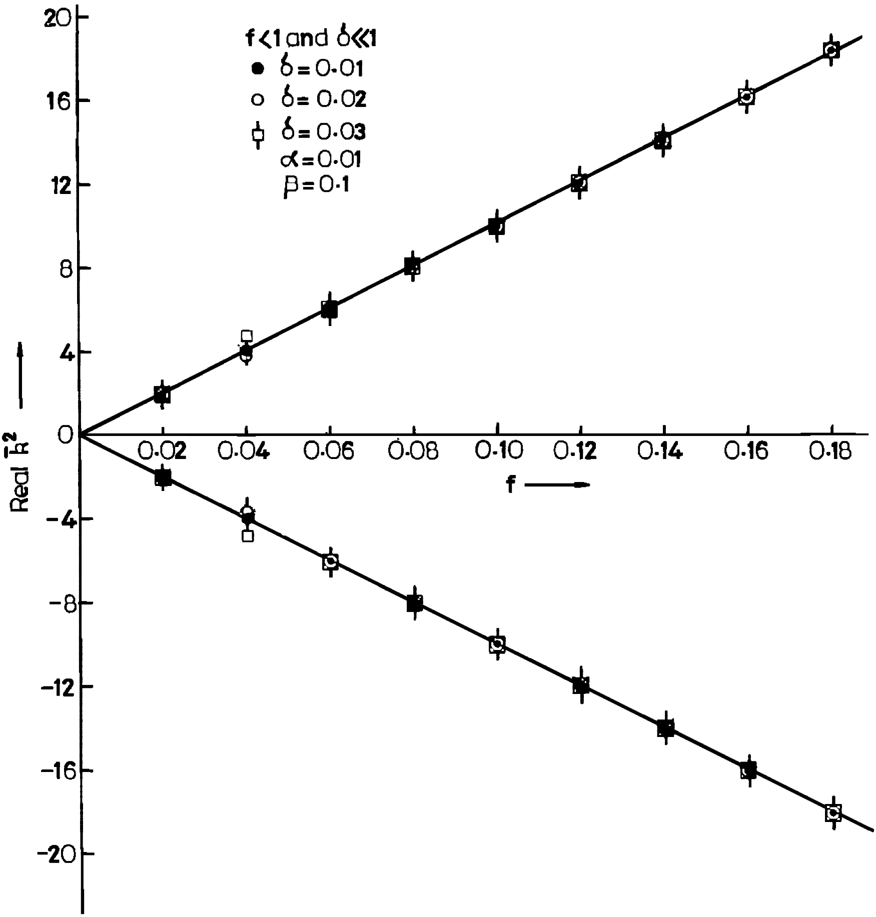


Fig. 4. Continued. (b) Variation of real $\bar{k}^2$ with f for $f = 0.02–0.18$, $\delta = 0.01–0.03$, $\alpha = 0.01$, and $\beta = 0.1$.

latter situation the wave shows a decay as it propagates through the dusty plasma. The different situations are picked out by the condition that Re $\bar{k}^2 < 0$ or Re $\bar{k}^2 > 0$, which imposes a restriction on $k_1^2 - k_2^2$. So we observe that the wave propagation in a dusty plasma has clearly distinctive features for the high- and low-frequency regions and also it behaves differently for low values (Hasegawa and Cheu, 1976; Cramer and Ponelly, 1981) and high values of δ, that is, for a more or a less dusty plasma.

Lastly, we can quote some analytical results which reproduce the above numerical observations correctly. From the dispersion relation for $f \gg 1$ or $f \ll 1$ we can compute the resonance and cutoff values. These are as follows.

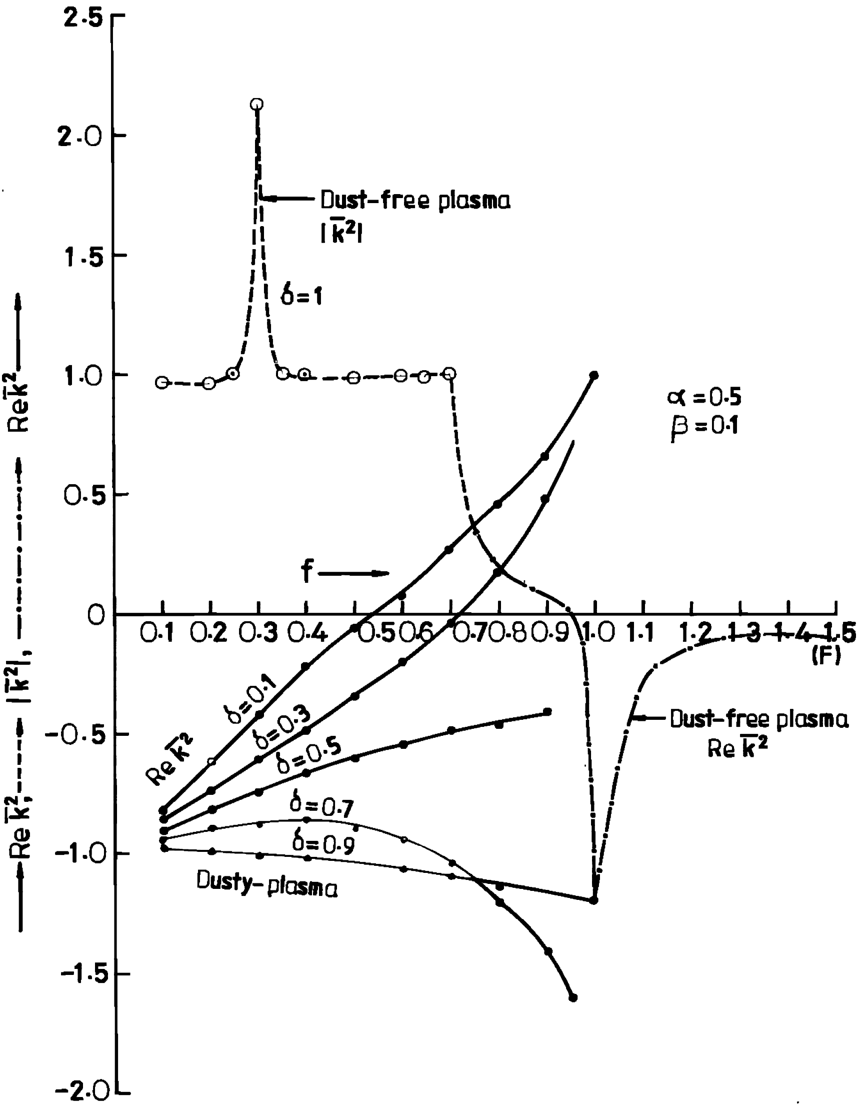


Fig. 4. Continued. (c) Variation of $|\bar{k}^2|$, real $\bar{k}^2$, with f for dusty and dust-free plasma, $f = 0.1–1.5$, $\alpha = 0.5$, and $\beta = 0.1$.

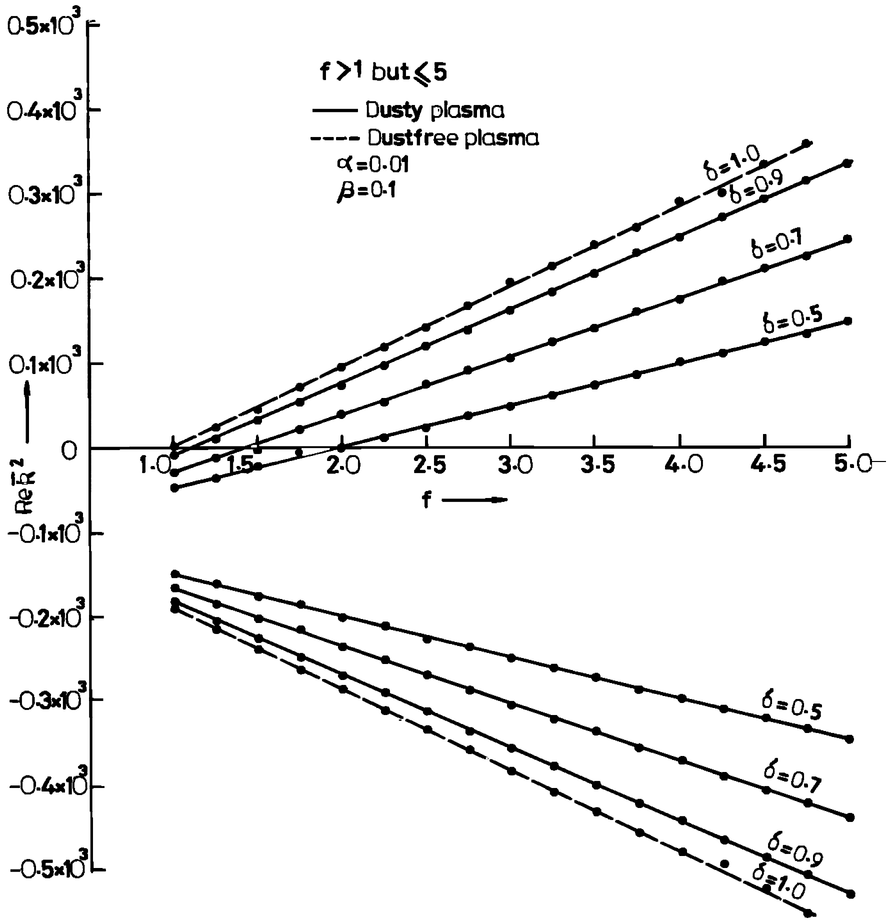


Fig. 5. Plot of $\bar{k}^2$ against f for different dust concentrations for (a) $f = 1.0–5.0$, $\delta = 0.5–0.9$, $\alpha = 0.01$, and $\beta = 0.1$; (b) $f = 5.0–50.0$, $\alpha = 0.01$, and $\beta = 0.1$; (c) $f = 1.0–18.0$, $\delta = 0.1–0.9$, $\alpha = 0.5$, and $\beta = 0.1$.

For $f \gg 1$ the cutoff frequency is given by $f^2 = 1 + [2 + \propto(1 - \beta\delta^2)]\alpha/\delta^2$, whereas for $f \ll 1$ the cutoff frequency is $f^2 = \alpha^2/(1 - \delta)^2$. The resonance frequency for $f \gg 1$ is $f^2 = 2\alpha\beta/\mu_0\varepsilon_0 V_s^2(1 + \beta\delta^2)$ and for $f \ll 1$, $f^2 = \alpha/\delta$. These are in conformity with the graphical depictions given in the figures.

4. CONCLUDING REMARKS

In the present paper, we derived the general dispersion relation and studied the propagation of waves in a magnetized dusty plasma for both low

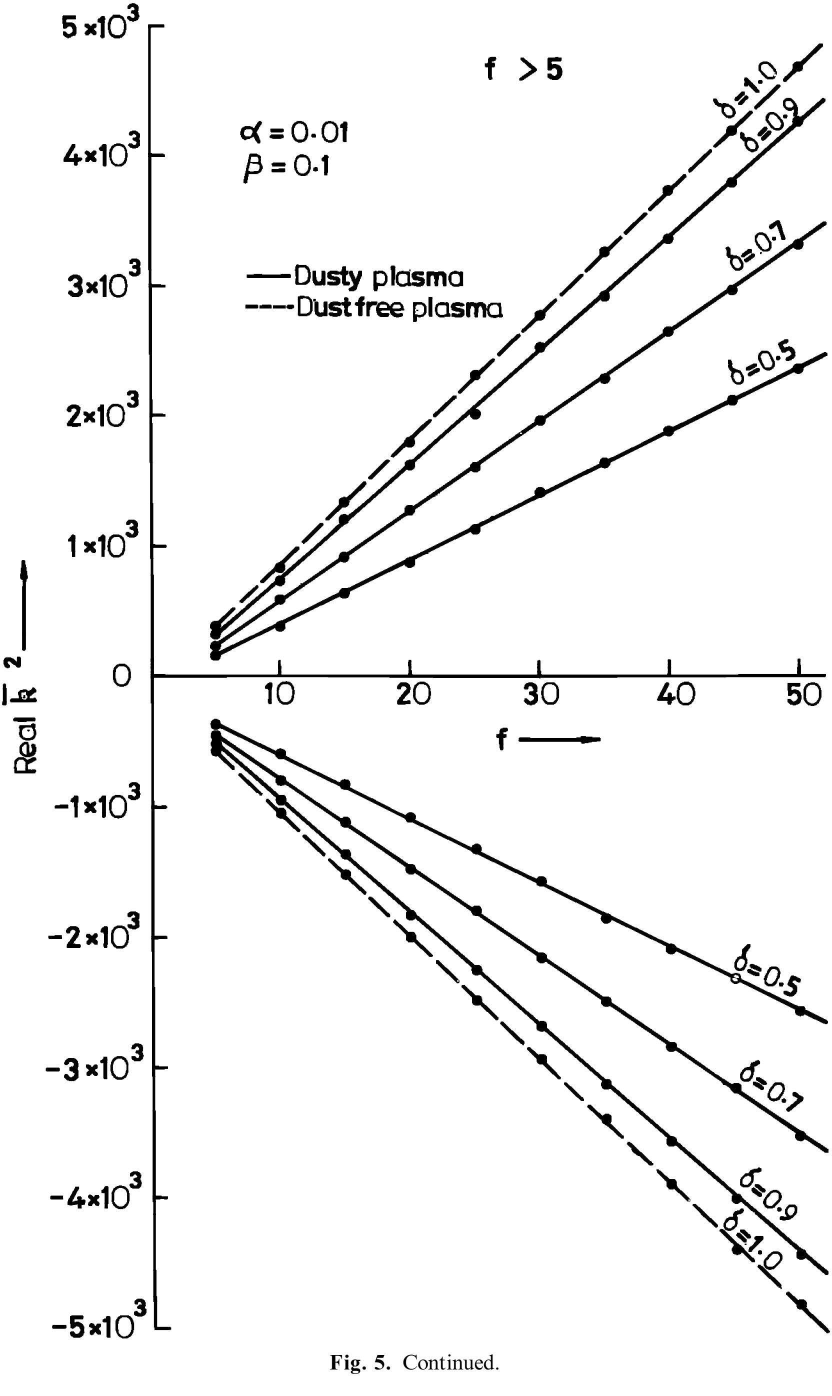


Fig. 5. Continued.

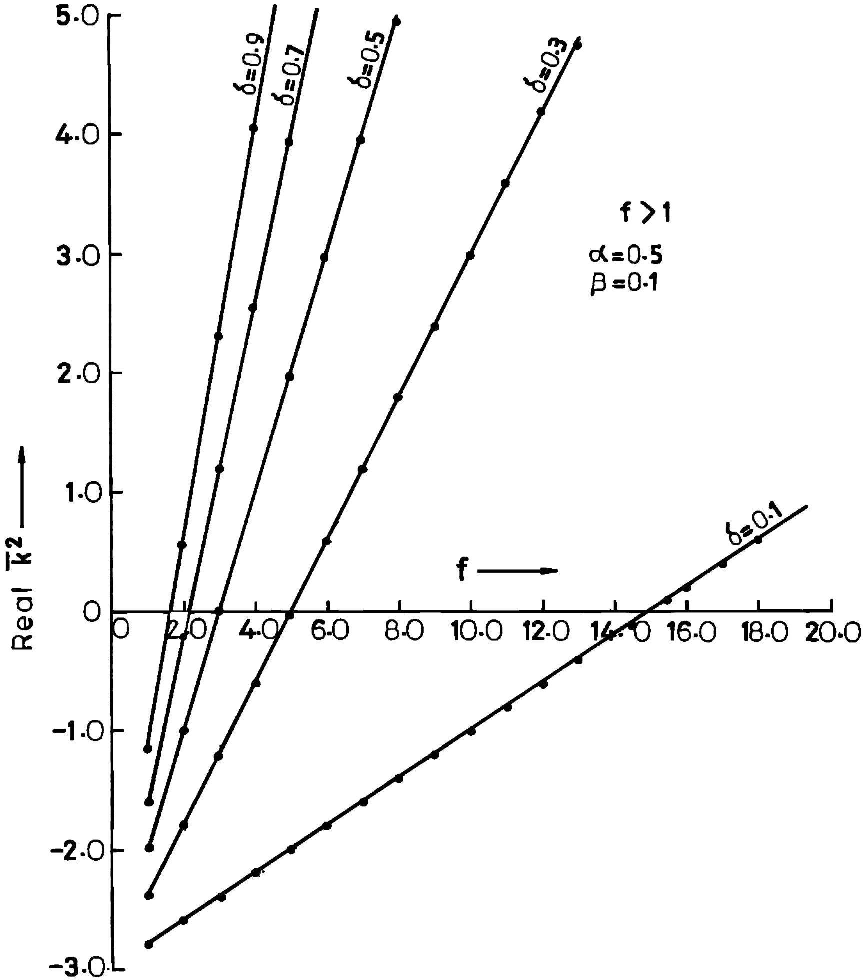


Fig. 5. Continued.

and high values of the ion-cyclotron frequency. We found that there are some errors in the values of the electric field components obtained by Cramer and Vladimirov (1996). In our analysis we assumed that the charge of the dust particles is constant. But the charged dust grains which are more massive than protons acquire high negative charges due to the preferential capture of electrons. Fluctuations in the grain charges due to the liberation or capture of additional electrons and protons translate as mass and momentum losses or gains, which can induce additional phenomena. Such fluctuations can change the stability criterion of electrostatic (Varma *et al.*, 1993: Jana *et al.*,

1993) and electromagnetic (Vladimirov, 1994a, b) waves in a dusty plasma (Verheest, and Menris, 1995). Due to this charge fluctuation, there is a collisional damping of acoustic waves (Dwivedi and Pandey, 1995). Moreover, the streaming of dust particles can excite waves in the low-frequency region (Haynes, 1988; Bharathram *et al.*, 1992; Kulkarni *et al.*, 1994). In a recent paper (Khurshed *et al.*, 1997), we investigated the effects of streaming electrons and ions on the propagation of waves in a magnetized dusty plasma. It is observed that the phase velocity of the waves for various modes is significantly affected by streaming electrons, giving new values of the cutoff frequency and resonance condition.

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